

## Unit Rates and Proportional Relationships

A **unit rate** is a relationship between two quantities that have different units with a denominator (divisor) of **one**.

Examples: miles per hour, cost per pound, cost per day

We use **division** to find a unit rate when given a total and asked to solve for one of something.

Clue words: per, each, an, in, every.

Usually the division is set up for us in the word problem:

_____	per	_____
First number	÷	Second number
Numerator	÷	Denominator
Dividend	÷	Divisor
Inside the box	÷	Outside the box
\$\$\$	÷	Time

A **proportional relationship** is one where the two quantities have a rate of change that is **constant** (equal). They form a proportion (two equal ratios/fractions).

Examples:  $\frac{1}{2} = \frac{2}{4}$     $\frac{10}{5} = \frac{12}{6} = \frac{2}{1}$

The **constant of proportionality** is the rate of change in a proportional relationship (the number you get when all the ratios are equal). We represent this value with the variable **k**. **k** is found by comparing the ratios in a table, graph, or ordered pairs.  $k = \frac{y}{x}$ . Think: What is happening to  $x$  to get  $y$ ? That's your  $k$ ! If  $y$  is smaller than  $x$ ,  $k$  should be a fraction.

A proportional relationship has the equation  **$y = kx$** . Substitute the value you found for  $k$  into the equation. You can check your equation by plugging in a value of  $x$ , multiplying it by  $k$ , and checking you get the corresponding  $y$  value.

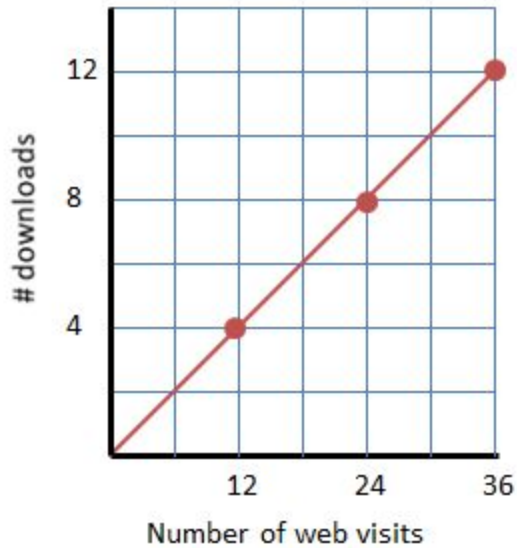
When you find the rate of change to check if a relationship is proportional from a table, you need to flip the table upside down!  $X$  is always the top row, or left column in a table.  $Y$  is always the bottom row, or right column in a table. Find the ratios  $\frac{y}{x}$  to get the constant of proportionality.

Examples:

<b>x</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>y</b>	<b>8</b>	<b>12</b>	<b>16</b>

$$k = \frac{y}{x} \quad \frac{8}{2} = \frac{12}{3} = \frac{16}{4} \text{ all ratios reduce to } \frac{4}{1} \text{ so } \mathbf{k = 4}$$

The equation will be  $y = \mathbf{4}x$  Check:  $8 = 4(2) \checkmark$



Use **REALLY NICE POINTS** to find the ratios.  
Remember every ordered pair is (x, y)  
x is the horizontal axis, y is the vertical axis

(12, 4), (24, 8), (36, 12)

$$k = \frac{y}{x} \quad \frac{4}{12} = \frac{8}{24} = \frac{12}{36} \quad \text{all ratios reduce to } \frac{1}{3} \quad \text{so } k = \frac{1}{3}$$

The equation will be  $y = \frac{1}{3}x$  Check:  $4 = \frac{1}{3}(12)$  ✓

As you can see, a proportional relationship forms a **straight line, passing through the origin.**

The constant of proportionality is the unit rate! When given a graph, if clearly visible, you can look where  $x = 1$ , and find the corresponding y value to find k! If a point is not clearly visible, use a **REALLY NICE POINT** and find k. Once you have an equation, plug in x values to find the corresponding y values.